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Optimization of circuits with two degrees of freedom controllers

Controllers with two degrees of freedom (2DOF) are not commonly used. From conventional controllers are different by front-end filter for setting desired value. They enable to achieve better dynamic properties for changes of desired value.

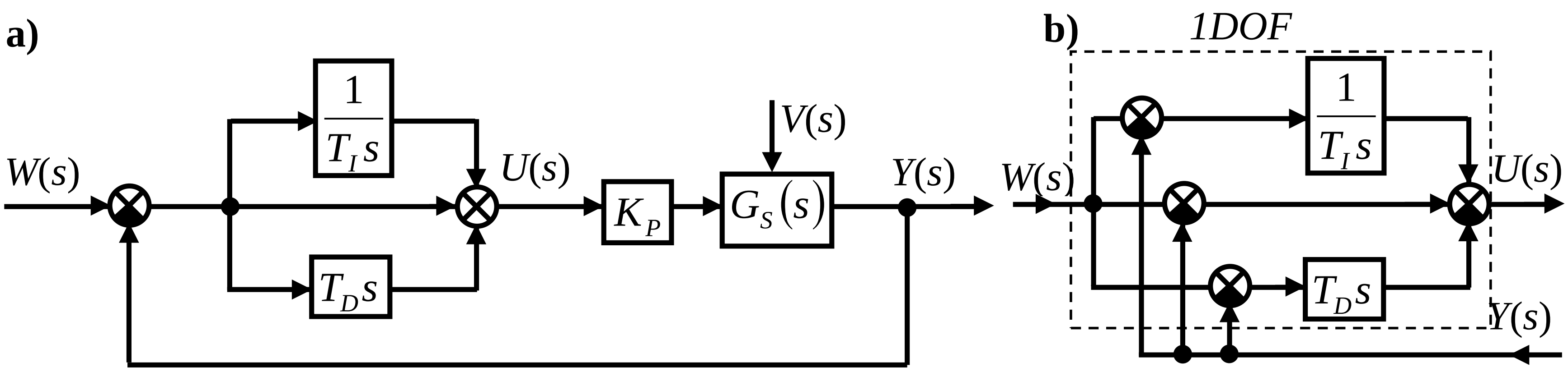

Fig. 1 *Simple controller PID with one degree of freedom*

Let's base it on simple controller circuit with one degree of freedom – fig.1.a.; Controller PID has all three members. Circuit can also be expressed in equivalent form in fig.1.b.

If we wish to evolve the solution to controller with two degrees of freedom (2DOF) we need to change weights of desired value for proportional and derivative members, as seen on Fig.2, the most common circuit variation [Åström, Hāgglund 1995].

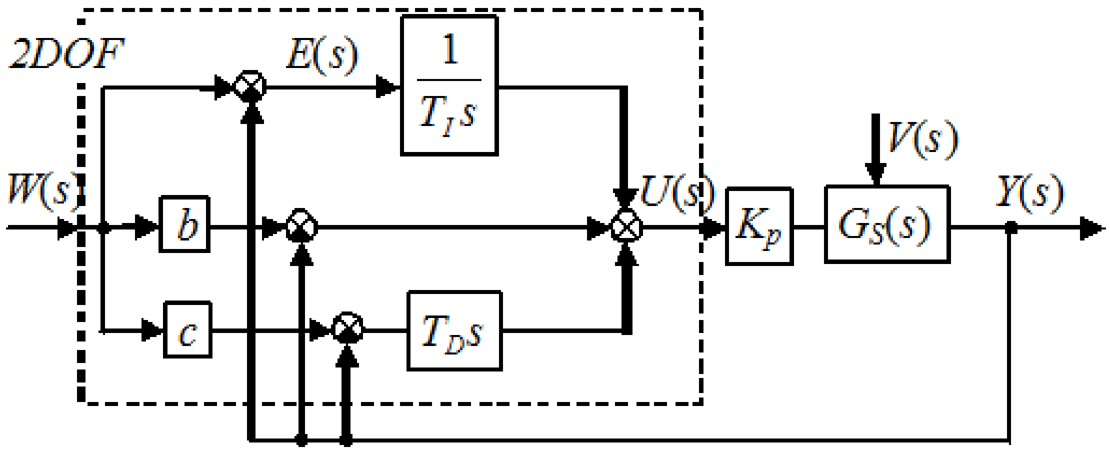

Fig. 2 Controller PID with two degrees of freedom – 2DOF

We see that with changes in error value $v(t)$, quality of control does not change, weights *b, c* are used only for changes of desired value $w(t)$. That is important property of controllers 2DOF - lower overshoot when desired value is changed and weights *b, c* do not affect control process with changes of error value. On input of integration member of controller must be control deviation $e(t)$ and therefore must be eventual weight for integration member $a = 1$. In case the controller would not contain integration member, control deviation must be on the input of proportional member and weight $b = 1$. In case $b = c = 1$ the controller is really a classical one degree of freedom controller as in Fig. 1.

The equation can be formulated as

$$U(s) = K_p \left\{ \left[bW(s) - Y(s) \right] + \frac{1}{T_I s} \left[W(s) - Y(s) \right] + T_D s \left[cW(s) - Y(s) \right] \right\} \quad (1)$$

$$U(s) = \underbrace{K_p \left(b + \frac{1}{T_I s} + cT_D s \right)}_{G_{ff}(s)} W(s) - \underbrace{K_p \left(1 + \frac{1}{T_I s} + T_D s \right)}_{G_R(s)} Y(s) \quad (2)$$

$$U(s) = G_{ff}(s) W(s) - G_R(s) Y(s) \quad (3)$$

and the last form of equation is equal to scheme in Fig.3.

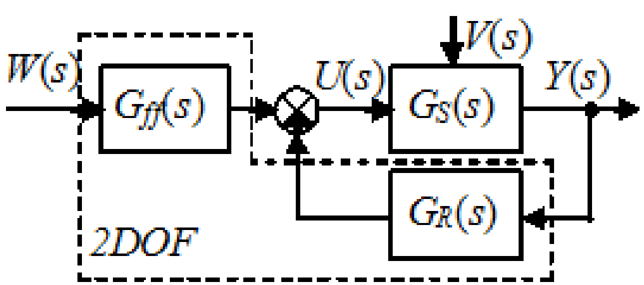

Fig. 3 Controller 2DOF as in form (3)

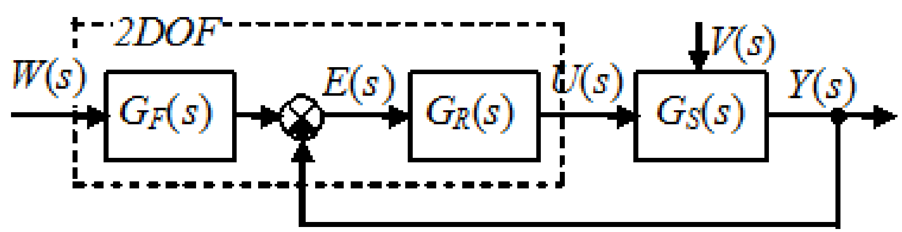

Fig. 4 Controller 2DOF as in form (5)

According to [Vítečková, Víteček 2011] equation (3) , where is

$$G_F(s) = \frac{G_{ff}(s)}{G_R(s)} = \frac{cT_D T_I s^2 + bT_I s + 1}{T_D T_I s^2 + T_I s + 1} \tag{4}$$

transfer of input filter of desired value *w.* Giving us equation

$$U(s) = G_F(s) G_R(s) W(s) - G_R(s) Y(s) \tag{5}$$

and it's corresponding scheme of 2DOF controller in Fig.4, mentioned in [Åström, Hāgglund 2006].

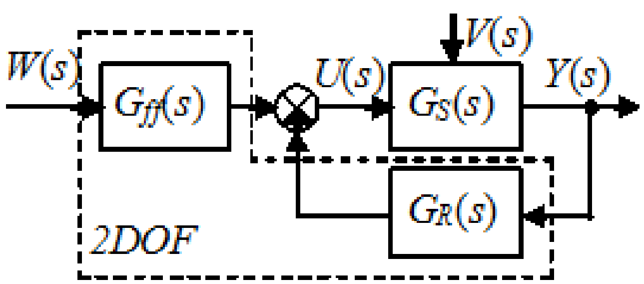

Fig. 3 Controller 2DOF as in form (3)

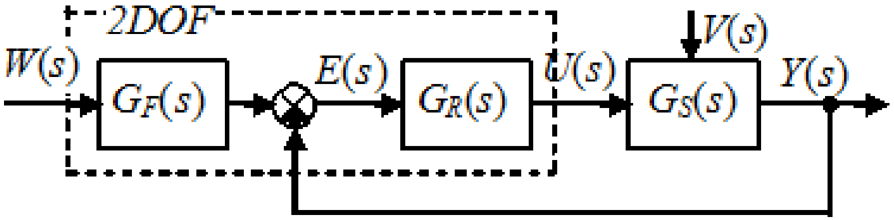

Fig. 4 Controller 2DOF as in form (5)

Quite similar circuit with controller 2DOF is mentioned in [Taguchi, Araki 2000], [Vítečková, Víteček 2011] with forward feedback of auxiliary controller – fig. 5

$$G_K(s) = K_P(\alpha' + \beta' T_D s) \tag{6}$$

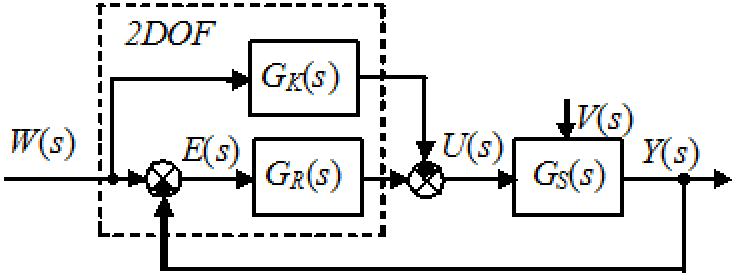

Fig. 5 *Controller 2DOF in form* (7)

In this circuit apply equation

$$U(s) = [G_R(s) - G_K(s)]W(s) - G_R(s)Y(s) \tag{7}$$

According to [Vítečková, Víteček 2011] it applies for

$$b = 1 - \alpha', \quad c = 1 - \beta' \tag{8}$$

That all mentioned circuit structures with controller 2DOF are mutually equivalent.

Basic function of controller PID 2DOF in in form (1) and according to Fig.2. If weights of desired value are $0 \leq b < 1$ and $0 \leq c < 1$, lowering of jump value of desired value *w*(*t*) occurs and therefore overshoot is lowered.

Simultaneously according to [Vítečková, Víteček 2011] most of the time response delay is **higher**.

By suitable choice of weights *b*, *c* can be achieved **lowering** of **overshoot** and also **adequate fast response**.